



Helge Holden  
Ragni Piene  
*Editors*



# The Abel Prize 2003–2007

The First Five Years



 Springer

# The Abel Prize



Niels Henrik Abel 1802–1829  
The only contemporary portrait of Abel, painted by Johan Gørbitz in 1826  
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Helge Holden • Ragni Piene  
Editors

# The Abel Prize

2003–2007 The First Five Years



Helge Holden  
Department of Mathematical Sciences  
Norwegian University  
of Science and Technology  
7491 Trondheim  
Norway  
holden@math.ntnu.no

Ragni Piene  
Centre of Mathematics for Applications  
and Department of Mathematics  
University of Oslo  
0316 Oslo  
Norway  
ragnip@math.uio.no

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# Preface

In 2002, the year marking the bicentennial of Abel's birth, the Norwegian Parliament established the Niels Henrik Abel Memorial Fund with the objective of creating an international prize for outstanding scientific work in the field of mathematics—the Abel Prize.

In this book we would like to present the Abel Prize and the Abel Laureates of the first five years. The book results from an initiative of the Mathematics section of the Norwegian Academy of Science and Letters. It is intended as the first volume in a series, each volume comprising five years.

The book starts with the history of the Abel Prize—a story that goes back more than a hundred years—written by Abel's biographer, Arild Stubhaug. It is followed by Nils A. Baas' biographical sketch of Atle Selberg; at the opening ceremony of the Abel Bicentennial Conference in Oslo in 2002, an Honorary Abel Prize was presented to Atle Selberg.

There is one part for each of the years 2003–2007. Each part starts with an autobiographical piece by the laureate(s). Then follows a text on the laureate's work: Pilar Bayer writes on the work of Jean-Pierre Serre, Nigel Hitchin on Atiyah–Singer's Index Theorem, Helge Holden and Peter Sarnak on the work of Peter Lax, Tom Körner on Lennart Carleson, and Terry Lyons on Srinivasa Varadhan. Each part contains a complete bibliography and a curriculum vitae, as well as photos—old and new.

The DVD included in the book contains the interviews that Martin Raussen and Christian Skau made with each laureate in connection with the Prize ceremonies in the years 2003–2007. Every year except the first, the interviews were broadcast on Norwegian national television. Transcripts of all interviews have been published in the *EMS Newsletter* and *Notices of the AMS*.

We would like to express our gratitude to the laureates for collaborating with us on this project, especially for providing the autobiographical pieces and the photos. We would like to thank the mathematicians who agreed to write about the laureates, and thus are helping us in making the laureates' work known to a broader audience.

Thanks go to Martin Raussen and Christian Skau for letting us use the interviews, to David Paukztello for his translations, to Marius Thaulé for his  $\text{\LaTeX}$  expertise and the preparation of the bibliographies, and to Anne-Marie Astad of the

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July 15, 2009

Helge Holden and Ragni Piene

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# The History of the Abel Prize

Arild Stubhaug

On the bicentennial of Niels Henrik Abel's birth in 2002, the Norwegian Government decided to establish a memorial fund of NOK 200 million. The chief purpose of the fund was to lay the financial groundwork for an annual international prize of NOK 6 million to one or more mathematicians for outstanding scientific work. The prize was awarded for the first time in 2003.

That is the history in brief of the Abel Prize as we know it today. Behind this government decision to commemorate and honor the country's great mathematician, however, lies a more than hundred year old wish and a short and intense period of activity.

Volumes of Abel's collected works were published in 1839 and 1881. The first was edited by Bernt Michael Holmboe (Abel's teacher), the second by Sophus Lie and Ludvig Sylow. Both editions were paid for with public funds and published to honor the famous scientist. The first time that there was a discussion in a broader context about honoring Niels Henrik Abel's memory, was at the meeting of Scandinavian natural scientists in Norway's capital in 1886. These meetings of natural scientists, which were held alternately in each of the Scandinavian capitals (with the exception of the very first meeting in 1839, which took place in Gothenburg, Sweden), were the most important fora for Scandinavian natural scientists. The meeting in 1886 in Oslo (called Christiania at the time) was the 13th in the series. At the meeting's farewell dinner, the Swedish mathematician Gösta Mittag-Leffler gave a toast in honor of Niels Henrik Abel, and he proposed starting a collection with the goal that in 16 years—in 1902, on the centennial of Abel's birth—a statue of the young genius could be erected. Money was collected during the meeting and national committees were appointed, but eventually the whole effort ran out of steam.

Mittag-Leffler, who had been publishing the Swedish mathematics journal, *Acta Mathematica*, since 1882, worked during these years to arrange and gather support for an international mathematics prize, namely King Oscar II's Mathematics Prize,

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A. Stubhaug (✉)

Matematisk institutt, Universitetet i Oslo, 0316 Oslo, Norway

e-mail: [arilds@math.uio.no](mailto:arilds@math.uio.no)

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a competition in which an answer was sought to one of four given questions. The prize was awarded on the King's 60th birthday in January 1889, and it was a tremendous success in every way. The prize winner was Henri Poincaré, who submitted a work that described chaos in space: a discovery that was only understood in its full breadth much later and that gradually developed into a major interdisciplinary research area. On the jury for the prize sat Charles Hermite and Karl Weierstrass together with Mittag-Leffler, and the latter discussed the possibility of establishing a permanent mathematics prize with King Oscar and various patrons and donors. Due to insufficient support, however, Mittag-Leffler initially tried to establish a smaller fund, and he proposed that money from this fund should be used for gold medals, which should be awarded to mathematicians who had published an exceptionally important work in *Acta Mathematica*. The gold medals were to be stamped with portraits of the greatest mathematicians, and he was of the opinion that it was suitable to begin with the greatest mathematician in the Nordic countries: Niels Henrik Abel.

These plans also came to naught. Instead, Mittag-Leffler managed to set up a fund that supported the editing of articles submitted to *Acta Mathematica* and that made it possible to invite great foreign mathematicians to Stockholm. When the content of Alfred Nobel's last will and testament became known in 1897, rumors abounded that Mittag-Leffler's financial antics and scientific plans and ideas might have dissuaded Nobel from providing funds for a prize in mathematics, in addition to those in physics, chemistry and medicine as well as literature and efforts to promote peace. It is true that Mittag-Leffler and Nobel discussed financial support for both Stockholm University College (now Stockholm University) and an extraordinary professorship for Sonja Kovalevsky and that they were in strong disagreement, but the reason why there was not any Nobel Prize in mathematics seems to clearly lie in Nobel's attitude to science and technology. He was a practical man and regarded mathematics in general as much too theoretical and having no practical applications.

The annual Nobel Prizes, awarded for the first time in 1901, quickly overshadowed other scientific prizes. At the academies of science in Paris and Berlin, mathematics prizes based on various problems, often in astronomy and navigation, had been awarded ever since the middle of the eighteenth century, and new prizes came into being in the nineteenth century [1]. Prizes were also announced in Leipzig, Göttingen and at other centers of learning. In 1897, the international Lobachevsky Prize was established at the University of Kazan. This prize was supposed to go to outstanding works in geometry, especially non-Euclidean geometry, and the first winner was Sophus Lie.

Sophus Lie, Norway's second world-class mathematician, died in 1899. One of the last things he used his international contact network for was to gather support for establishing a fund that would award an Abel Prize every fifth year for outstanding work in pure mathematics. Apparently, an inspiration in Lie's work was precisely the fact that Nobel's plans included no prize in mathematics. From leading centers of mathematical learning, Sophus Lie received overwhelming support for such an Abel Prize in the spring of 1898. From Rome and Pisa came assurances of support from Luigi Cremona and Luigi Bianchi; from Paris Émile Picard wrote that

both he and Hermite would donate money to the fund, and Picard, who otherwise would like to see a more frequent awarding of the prize than once every fifth year, reported that through its universities and lyceums France would also probably be able to contribute large sums; Gaston Darboux followed up with similar positive reactions and thought that all mathematicians in the Academy of Science in Paris would support an Abel Fund; Sophus Lie also received a warm declaration of support from A.R. Forsyth at Cambridge, who thought that Lord Kelvin would certainly lend his support to the fund; Felix Klein at Göttingen reported that he would obviously support the work, and he believed that David Hilbert would do so as well; Lazarus Fuchs was also supportive. The only mathematicians who expressed skepticism were Georg Frobenius and H.A. Schwarz in Berlin; they thought prizes in general often diverted younger talents away from the true scientific path.

Sophus Lie's contacts and promises of support, however, were related to him personally. When Sophus Lie died, there was no one else who could carry on the work.

At the celebration of the centennial of Abel's birth in 1902, three main tasks were formulated in Norwegian political and scientific circles: first, to arrange a broad cultural commemoration, second, to erect a worthy monument to the genius, and third, to establish an international Abel Prize. The first two tasks were achieved. The Abel commemoration in September 1902 was held with pomp and circumstance, and students, citizens, scientists, artists, the national assembly, the government and the Royal House all took part. A number of foreign mathematicians were present and were awarded honorary doctorates. Gustav Vigeland's great Abel Monument on the Royal Palace grounds (in Oslo) was unveiled six years later, but the plans for an Abel Prize were put on ice for reasons of national politics.



Gustav Vigeland's Abel Monument, Oslo