

**Heat and  
Mass Transfer**

**N. Kockmann**

# Transport Phenomena in Micro Process Engineering



**Springer**

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## *Heat and Mass Transfer*

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*Series Editors: D. Mewes and F. Mayinger*

Norbert Kockmann

# **Transport Phenomena in Micro Process Engineering**

With 214 Figures and 17 Tables



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for Eva Christina, Paul Joey, and Lilly Sophie ...

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## Preface

Transport phenomena and kinetic processes are found in almost every technical application where mass or energy are transformed into other shapes and forms. Characteristic time and length scales determine process characteristics and their interaction. The unified notion of transport phenomena was introduced by Bird, Stuart, and Lightfoot in their well-known textbook [1, 2] and has a broad influence in many engineering areas. With miniaturization of process equipment, the characteristic length and time scales shift to different regimes, where they are beneficial for enhanced applications, see [3]. Conversely, they may prevent conventional process conditions, which must be reassessed. The transport phenomena in microstructures treated here concern momentum, species, mass, and energy transfer combined with entropy generation, reaction kinetics, and coupled processes. Almost all transport processes dealt with are in the continuum range and can be described by linear correlations, however, the limitations and exceptions are given. The length scale of transport processes reaches from molecular scale of gas scattering to complex turbulent flow in the millimeter range.

The present work is outlined according to the nature of the transport phenomena and kinetic processes. The first two chapters introduce the scope of micro process engineering and general process engineering to the unfamiliar reader. The second chapter also presents the fundamentals of transport processes from single molecular encounters to transport in multiphase flow. Engineering tools, such as dimensional analysis or order-of-magnitude estimations are explained, which allow treatment of complex nonlinear systems.

The third and fourth chapters cover momentum and heat transfer in microchannels with emphasis on laminar convection and the design of appropriate channel geometries. Convective transport enhancement is employed for mixing enhancement in microchannels, described in Chapter 5. Aside from the appropriate characterization of the mixing process and device, typical properties of convective laminar micromixers demonstrate their excellent performance with rapid, mixing-sensitive chemical reactions for increased selectivity or fast particle precipitation, described in Chapter 6.

Chapter 7 is unique in the field of micro process engineering topics and deals with coupled transport processes. Micro fabrication technology enables tailor-made geometries with appropriate materials to augment coupled transport phenomena. This is used for efficient generation of electrical energy by thermoelectric devices or single-phase separation by thermodiffusion. In spite of the advances presented, there is still a lot of effort required to improve devices and integrate them into suitable applications. Some suggestions for this ongoing process are referenced and summarized at the end of each chapter, as well as in Chapter 8.

This work may also be a contribution to the tenth anniversary of the IMRET conference [4] on micro reaction technology, first held in 1997 in Frankfurt, Germany. It might also be regarded as the twentieth anniversary of the first patent on microstructured devices for process engineering in the former Eastern Germany [5]. Finally, the first textbook on microchemical technology is more than one hundred years old [6, 7], and the first literature on microscope usage in chemical analysis appeared approx. 130 years ago [8]. My own first contact with micro process engineering, observing technical processes under the microscope, was a diffusion cell in crystallization, in which a doctorate colleague observed diffusion processes in crystal layers and inclusions under a temperature gradient. This concept of an observation cell was very effective and is similar to many of the concepts presented here.

Whilst writing this work, the idea of the “rhizome” introduced by Deleuze [9] often came to mind. Originally indicating the fine network of plant roots, Deleuze and Guattari [10] presented their philosophical concept to indicate a living, interwoven network, cross-linking many disciplines and fields, and mirroring the modern, complex world. The topic of this thesis is just a small area in science and engineering, but embraces and influences so many disciplines and fields that this idea may also apply here. The combination of different disciplines presents new opportunities for many fields in microfluidics and process intensification. I hope the reader is not affronted by the complexity of micro process engineering, but will take this work as a guideline for tackling future challenges. Assisting this process, Kornwachs stated [11] that engineers, while building machines, shape the technical landscape for actual necessities. Hence, engineers are becoming system designers and creators of culture, opening a new role, which they must learn to use.

This book is the outcome of many diploma works, a dissertation, many group discussions, and the collected effort of many contributors from the University of Freiburg, Department of Microsystems Engineering – IMTEK, other universities, research labs, and from the “real world”. Thanks for the good atmosphere, for your encouraging support and many helpful discussions. The text was carefully proof-read by Paul Thomas who is not responsible for any remaining faults.

Finally, very special thanks go to my family, for their enduring care, support, and love throughout the years.

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## Nomenclature

### Latin Letters

Variable	Denotation	Unit
$A$	(surface) area or cross section	$\text{m}^2$
$a$	activity	—
$a$	temperature conductivity $a = \frac{\lambda}{\rho c_p}$	$\text{m}^2/\text{s}$
$a_V$	specific surface $= A/V$	$\text{m}^2/\text{m}^3$
$B_{hom}$	rate of homogeneous nucleation	$1/\text{m}^3\text{s}^{-1}$
$b$	geometrical factor, channel width	$\text{m}$
$C$	constant	—
$C_f$	friction factor coefficient for laminar duct flow	—
$C_i$	ratio of heat capacity fluxes	—
$c$	specific velocity	$\text{m}/\text{s}$
$c$	molecular velocity	$\text{m}/\text{s}$
$c$	speed of sound	$\text{m}/\text{s}$
$c_i$	concentration of component i	—
$c_p$	isobaric specific heat capacity	$\text{J}/\text{kg K}$
$c_v$	isochoric specific heat capacity	$\text{J}/\text{kg K}$
$D$	diffusivity, diffusion coefficient	$\text{m}^2/\text{s}$
$D$	diameter	$\text{m}$
$d_P$	particle diameter	$\text{m}$
$d_h$	hydraulic diameter $= \frac{4 \cdot \text{cross section area}}{\text{wetted perimeter}}$	$\text{m}$
$E$	energy	$\text{J}$
$E_{kin}$	kinetic energy	$\text{J}$
$E_{pot}$	potential energy	$\text{J}$
$e$	specific energy	$\text{J}/\text{kg}$
$F$	force	$\text{N}$
$F$	cumulative number distribution (PSD)	—

Variable	Denotation	Unit
$f$	probability distribution function	—
$f$	number fraction frequency (PSD)	—
$G$	mass velocity = $\dot{m}/A$ (US literature)	kg/m <sup>2</sup> s
$G$	linear growth rate (PBE)	m/s
$g$	gravity constant	m/s <sup>2</sup>
$g$	temperature jump coefficient	—
$H$	enthalpy	J
$\Delta H_R$	reaction enthalpy	J/mol <sup>3</sup>
$\Delta h_R$	specific reaction enthalpy	J/m <sup>3</sup>
$H_i$	Henry coefficient	Pa
$h$	specific enthalpy	J/kg
$\Delta h_V$	latent heat of vaporization	J/kg
$h$	heat transfer coefficient (US literature)	W/m <sup>2</sup> K
$h$	geometrical factor, height	m
$I$	electrical current	A
$J$	general energy current	J/s
$K$	general coefficient	—
$k$	overall heat transfer coefficient	W/m <sup>2</sup> K
$k$	reaction rate constant for reaction of order $m$	1/s
$k$	Boltzmann constant ( $1.380662 \cdot 10^{-23}$ )	J/K
$L$	length	m
$L_{ij}$	general transport coefficient	—
$L_p$	rate of production	mol/s
$l$	length, characteristic	m
$l_C$	channel length	m
$l_m$	characteristic mixing length	m
$l_P$	length of the wetted perimeter	m
$M$	molar mass	kg/mol
$M_m$	mass of a single atom or molecule	kg
$m$	mass	kg
$\dot{m}$	mass flow rate	kg/s
$m$	molality	mol/kg
$m$	reaction order	—
$N$	number	—
$N_i$	ratio of transferred heat to heat capacity, number of transfer units NTU	—
$n$	amount of substance	mol
$\dot{n}$	molar flow rate	mol/s
$n$	number density concentration	1/m <sup>4</sup>
$n$	rotation speed	1/s
$P$	power	W
$p$	pressure	bar, Pa = N/m <sup>2</sup>

Variable	Denotation	Unit
$p_i$	partial pressure	bar, Pa = N/m <sup>2</sup>
$Q$	heat	J = Nm = kg/m <sup>2</sup> s <sup>2</sup>
$\dot{Q}$	heat flux	W = J/s
$q$	specific heat	J/kg
$\dot{q}$	specific heat flux	W/m <sup>2</sup>
$R$	radius	m
$R$	radius of spherical particles	m
$R$	individual gas constant, $= R_m/M$	J/kgK
$R$	electrical resistance	$\Omega$
$R_i$	transformation rate	mol/m <sup>3</sup> s
$R_m$	universal gas constant (8.314 J/mol K)	J/mol K
$r$	spherical coordinate (radius)	m
$r$	reaction rate	1/s
$S$	entropy	J/K
$S$	selectivity (competitive reactions)	—
$S_a$	activity-based supersaturation	—
$S_c$	concentration-based supersaturation	—
$S'$	heat production potential	—
$s$	specific entropy	J/kg K
$T$	temperature	K, °C
$\Delta T_L$	spatial temperature gradient	K/m
$\Delta T_t$	temporal temperature gradient	K/s
$t$	time	s
$t_m$	characteristic mixing time	s
$t_{m90}$	characteristic mixing time with 90 % mixing quality	s
$t_R$	characteristic reaction time	s
$t_P$	mean residence time	s
$U$	inner energy	J
$U$	electric voltage	V
$u$	specific inner energy	J/kg
$u$	velocity in x-direction	m/s
$V$	volume	m <sup>3</sup>
$\dot{V}$	volume flow rate	m <sup>3</sup> /s
$v$	velocity in y-direction	m/s
$v$	specific volume	m <sup>3</sup> /kg
$W$	work	J
$W_t$	technical work	J
$W_{diss}$	dissipated work or energy	J
$w$	width of phase lamellae	m
$w$	velocity in z-direction	m/s
$\vec{w}$	vector of velocity ( $u, v, w$ )	m/s