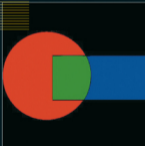


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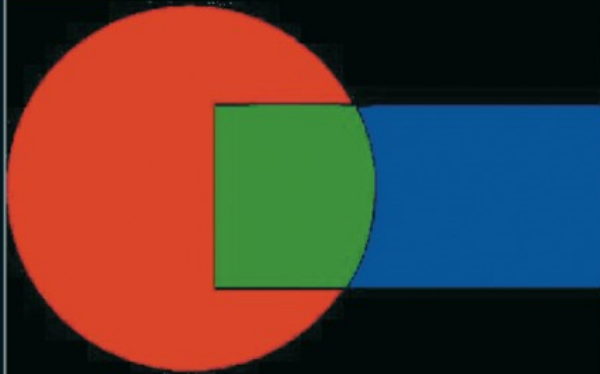
# Domain Decomposition Methods in Science and Engineering



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# Domain Decomposition Methods in Science and Engineering

With 184 Figures

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## Editors

Ralf Kornhuber  
Fachbereich Mathematik und  
Informatik  
Freie Universität Berlin  
Arnimallee 2-6  
14195 Berlin, Germany  
e-mail: kornhuber@math.fu-berlin.de

Ronald Hoppe  
Department of Mathematics  
University of Houston  
Philip G. Hoffman Hall 651  
77204-3008 Houston, TX, USA  
e-mail: rohop@math.uh.edu

Jacques Périaux  
Dassault Aviation  
78 Quai Marcel Dassault  
Cedex 300  
92552 St. Cloud, France  
e-mail:  
jacques.periaux@dassault-aviation.fr

Olivier Pironneau  
Laboratoire Jacques-Louis Lions  
Université Paris VI  
175 Rue de Chevaleret  
75013 Paris, France  
e-mail:  
olivier.pironneau@ann.jussieu.fr

Olof Widlund  
Courant Institute of Mathematical  
Science  
New York University  
Mercer Street 251  
10012 New York, USA  
e-mail: widlund@cims.nyu.edu

Jinchao Xu  
Department of Mathematics  
Eberly College of Science  
Pennsylvania State University  
McAllister Building 218  
16802 University Park, PA, USA  
e-mail: xu@math.psu.edu

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## Preface

This volume contains a selection of 72 papers presented at the 15<sup>th</sup> International Conference on Domain Decomposition which was hosted by Freie Universität Berlin (FU) in cooperation with Zuse Institute Berlin (ZIB), Weierstrass Institute Berlin (WIAS) and the DFG Research Center ‘Mathematics for Key Technologies’ in Berlin, Germany, July 21 - 25, 2003. The attendance of 167 scientists from 24 countries accentuates the relevance of this series of almost annual meetings. In addition, an introductory tutorial by William D. Gropp and David E. Keyes arranged in the run up to the conference attracted 31 participants from all parts of the world, most of which were students. The conference itself included 15 plenary lectures delivered by leading experts in the field, 12 Minisymposia, 37 contributed talks and a poster session. A total of 144 presentations made this meeting one of the largest in the series of domain decomposition conferences. Since three parallel sessions were employed in order to accommodate as many presenters as possible, attendees and non-attendees alike may turn to this volume to keep up with future trends that might be guessed from the diversity of subjects.

Domain decomposition conferences have become the most important market place world wide for exchanging and discussing new ideas about the old algorithmic paradigm of ‘divide and conquer’. Much of this reputation stems from the close interaction of experts in numerical analysis and practitioners from various fields of application concerning *fast and reliable iterative methods* for discretized partial differential equations: Schwarz methods and substructuring techniques form today’s basis for large scale parallel computing. The unified view on the decomposition into subdomains and the decomposition into frequencies in terms of abstract Schwarz methods or subspace correction bridged the gap between domain decomposition and multigrid. Sophisticated finite element tearing and interconnecting techniques opened new perspectives (not only) in linear elasticity.

While classical domain decomposition concentrates on a given discretized PDE, coupling/decoupling techniques have meanwhile been applied successfully to derive efficient solution procedures including the *discretization* itself:

Mortar finite elements are most famous for their flexibility, e.g., with respect to non-matching grids, a property which is particularly attractive in multi-body contact. Other promising results concern the fast solution of time-dependent problems by waveform relaxations with optimized coupling conditions or by parareal algorithms.

The two latter approaches are motivated by parallel computation. On the other hand, it is the underlying physical background that motivates, e.g., the splitting of problems on an unbounded domain into a bounded and an unbounded part and gives rise to different discretizations in these subdomains together with suitable coupling conditions. Many other physical problems involve the localisation of the physics and their transient variability across the geometric domain. For the mathematical description of such heterogeneous processes it is important to understand various options of coupling subdomains in relation to the overall multi-physics problem. In this way, heterogeneous domain decomposition can be regarded as a new and promising approach to the *mathematical modeling* of complex phenomena on multiple scales.

This volume reviews recent developments in mathematical modeling, discretization, and fast and reliable solution by domain decomposition or related techniques, including implementation issues. Applications comprise biocomputing, computational mechanics, combustion, electromagnetics, electronic packaging, electrodynamics, fluid dynamics, medicine, metallurgy, microwave technology, optimal control, porous media flow, and voice generation. For the convenience of readers coming recently into the subject, a bibliography of previous proceedings is provided below, along with some major recent review articles and related special interest volumes. This list will inevitably be found embarrassingly incomplete. (No attempt has been made to supplement this list with the larger and closely related literature of multigrid and general iterative methods, except for the books by Hackbusch and Saad, which have significant domain decomposition components.)

- P. Bjørstad, M. Espedal, and D. Keyes, editors. *Proc. Ninth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Ullensvang, 1997. Wiley, New York, 1999.
- T. Chan, R. Glowinski, J. Périaux, and O. Widlund, editors. *Proc. Second Int. Symp. on Domain Decomposition Methods for Partial Differential Equations*, Los Angeles, 1988. SIAM, Philadelphia, 1989.
- T. Chan, R. Glowinski, J. Périaux, and O. Widlund, editors. *Proc. Third Int. Symp. on Domain Decomposition Methods for Partial Differential Equations*, Houston, 1989. SIAM, Philadelphia, 1990.
- T. Chan, T. Kako, H. Kawarada, and O. Pironneau, editors. *Proc. Twelfth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Chiba, 1999. DDM.org, Bergen, 2001.

- T. Chan and T. Mathew. Domain decomposition algorithms. *Acta Numerica*, pages 61–143, 1994.
- M. Débit, M. Garbey, R. Hoppe, D. Keyes, Y. Kuznetsov, and J. Périaux, editors. *Proc. Thirteenth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Lyon, 2000. CINME, Barcelona, 2002.
- C. Farhat and F.-X. Roux. Implicit parallel processing in structural mechanics. *Computational Mechanics Advances*, 2:1–124, 1994.
- R. Glowinski, G. Golub, G. Meurant, and J. Périaux, editors. *Proc. First Int. Symp. on Domain Decomposition Methods for Partial Differential Equations*, Paris, 1987. SIAM, Philadelphia, 1988.
- R. Glowinski, Y. Kuznetsov, G. Meurant, J. Périaux, and O. Widlund, editors. *Proc. Fourth Int. Symp. on Domain Decomposition Methods for Partial Differential Equations*, Moscow, 1990. SIAM, Philadelphia, 1991.
- R. Glowinski, J. Périaux, Z.-C. Shi, and O. Widlund, editors. *Proc. Eighth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Beijing, 1995. Wiley, Strasbourg, 1997.
- W. Hackbusch. *Iterative Methods for Large Sparse Linear Systems*. Springer, Heidelberg, 1993.
- I. Herrera, D. Keyes, O. Widlund, and R. Yates, editors. *Proc. Fourteenth Int. Conf. on Domain Decomposition Methods in Science and Engineering*, Cocoyoc, 2002. UNAM, Mexico City, 2003.
- D. Keyes, T. Chan, G. Meurant, J. Scroggs, and R. Voigt, editors. *Proc. Fifth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Norfolk, 1991. SIAM, Philadelphia, 1992.
- D. Keyes, Y. Saad, and D. Truhlar, editors. *Domain-based Parallelism and Problem Decomposition Methods in Science and Engineering*, 1995. SIAM, Philadelphia.
- D. Keyes and J. Xu, editors. *Proc. Seventh Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, PennState, 1993. AMS, Providence, 1995.
- B. Khoromskij and G. Wittum. *Numerical Solution of Elliptic Differential Equations by Reduction to the Interface*. Springer, 2004.
- C.-H. Lai, P. Bjørstad, M. Cross, and O. Widlund, editors. *Proc. Eleventh Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Greenwich, 1999. DDM.org, Bergen, 2000.
- P. Le Tallec. Domain decomposition methods in computational mechanics. *Computational Mechanics Advances*, 2:121–220, 1994.
- J. Mandel, C. Farhat, and X.-C. Cai, editors. *Proc. Tenth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Boulder, 1998. AMS, Providence, 1999.
- L. Pavarino and A. Toselli. *Recent Developments in Domain Decomposition Methods*, volume 23 of *Lecture Notes in Computational Science & Engineering*. Springer, 2002.



- A. Quarteroni, J. Périaux, Y. Kuznetsov, and O. Widlund, editors. *Proc. Sixth Int. Conf. on Domain Decomposition Methods for Partial Differential Equations*, Como, 1992. AMS, Providence, 1994.
- A. Quarteroni and A. Valli. *Domain Decomposition Methods for Partial Differential Equations*. Oxford, 1999.
- Y. Saad. *Iterative Methods for Sparse Linear Systems*. PWS, Boston, 1996.
- B. Smith, P. Bjørstad, and W. Gropp. *Domain Decomposition: Parallel Multilevel Algorithms for Elliptic Partial Differential Equations*. Cambridge Univ. Press, Cambridge, 1996.
- A. Toselli and O. Widlund. *Domain Decomposition Methods*. Springer, 2004.
- B. Wohlmuth. *Discretization Methods and Iterative Solvers on Domain Decomposition*. Springer, 2001.
- J. Xu. Iterative methods by space decomposition and subspace correction. *SIAM Review*, 34:581–613, 1991.

We also recommend the homepage for domain decomposition on the World Wide Web [www.ddm.org](http://www.ddm.org) maintained by Martin Gander. This site features links to past and future conferences, a growing number of conference proceedings together with updated bibliographic and personal information pertaining to domain decomposition.

We wish to thank all members of the Scientific Committee for Domain Decomposition Conferences, and in particular the chair Ronald H.W. Hoppe, for their help in setting the scientific direction of this conference. We are also grateful to the organizers of the minisymposia for shaping the profile of the scientific program and attracting high-quality presentations. The conference offered a fruitful integration of scientific excellence of speakers with a great level of interaction not only during the sessions but also along the friendly conference dinner under the ‘communication tent’, bringing a pleasant and relaxed atmosphere for exchanging information among attendees and lecturers. The local organization was carried out by a wonderful team of almost 50 members of FU Berlin, Zuse Institute Berlin (ZIB), and Weierstrass Institute Berlin (WIAS). We thank all members of the local organizing committee chaired by Ralf Kornhuber and, most notably, the conference manager Sabrina Nordt for perfectly taking care of all aspects of preparing and running DD15.

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**Ralf Kornhuber**

Berlin, Germany

**Ronald H.W. Hoppe**

Augsburg, Germany and Houston, USA

**Jacques Périaux**

Paris, France

**Olivier Pironneau**

Paris, France

**Olof B. Widlund**

New York, USA

**Jinchao Xu**

PennState, USA

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