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CISM International Centre for Mechanical Sciences

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INTERNATIONAL CENTRE FOR MECHANICAL SCIENCES

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DYNAMIC METHODS FOR DAMAGE DETECTION IN STRUCTURES

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PREFACE

Non-destructive testing aimed at monitoring, structural identification and diagnostics is of strategic importance in many branches of civil and mechanical engineering. This type of tests is widely practiced and directly affects topical issues regarding the design of new buildings and the repair and monitoring of existing ones. The load-bearing capacity of a structure can now be evaluated using well-established mechanical modelling methods aided by computing facilities of great capability. However, to ensure reliable results, models must be calibrated with accurate information on the characteristics of materials and structural components. To this end, non-destructive techniques are a useful tool from several points of view. Particularly, by measuring structural response, they provide guidance on the validation of structural descriptions or of the mathematical models of material behaviour.

Diagnostic engineering is a crucial area for the application of non-destructive testing methods. Repeated tests over time can indicate the emergence of possible damage occurring during the structure's lifetime and provide quantitative estimates of the level of residual safety.

Of the many non-destructive testing techniques now available, dynamic methods enjoy growing focus among the engineering community. Conventional diagnostic methods, such as those based on visual inspection, thermal or ultrasonic analysis, are local by nature. To be effective these require direct accessibility of the region to be inspected and a good preliminary knowledge of the position of the defective area. Techniques based on the study of the dynamic response of the structure or wave propagation, on the contrary, are a potentially effective diagnostic tool. These can operate on a global scale and do not require a priori information on the damaged area.

Recent technological progress has generated extremely accurate and reliable experimental methods, enabling a good estimate of changes in the dynamic behaviour of a structural system caused by possible damage. Although experimental techniques are now well-established, the interpretation of measurements still lags somewhat behind. This particularly concerns identification and structural diagnostics due to their nature of inverse problems. Indeed, in these applications one wishes to determine some mechanical properties of a system on the basis of measurements of its response, in part exchanging the role of the unknowns and data compared to the direct problems of structural analysis.

Hence, concerns typical of inverse problems arise, such as high nonlinearity, non-uniqueness or non-continuous dependence of the solution on the data. When identification techniques are applied to the study of real-world structures, *additional obstacles arise given the complexity of structural modelling, the inaccuracy of the analytical models used to interpret experiments, measurement errors and incomplete field data. Furthermore, the results of the theoretical mathematical formulation of problems of identification and diagnostics, given the present state-of-knowledge in the field, focus on quality, while practical needs often require more specific and quantitative estimates of quantities to be identified. To overcome these obstacles, standard procedures often do not suffice and an individual approach must be applied to tackle the intrinsic nature of the problem, using specific experimental, theoretical and numerical methods. It is for these reasons that use of damage identification techniques still involves delicate issues that are only now being clarified in international scientific literature.*

The CISM Course "Dynamic Methods for Damage Detection in Structures" was an opportunity to present an updated state-of-the-art overview. The aim was to tackle both theoretical and experimental aspects of dynamic non-destructive methods, with special emphasis on advanced research in the field today.

The opening chapter by Vestroni and Pau describes basic concepts for the dynamic characterization of discrete vibrating systems. Chapter 2, by Friswell, gives an overview of the use of inverse methods in damage detection and location, using measured vibration data. Regularisation techniques to reduce ill-conditioning effects are presented and problems discussed relating to the inverse approach to structural health monitoring, such as modelling errors, environmental effects, damage models and sensor validation. Chapter 3, by Betti, presents a methodology to identify mass, stiffness and damping coefficients of a discrete vibrating system based on the measurement of input/output time histories. Using this approach, structural damage can be assessed by comparing the undamaged and damaged estimates of the physical parameters. Cases of partial/limited instrumentation and the effect of model reduction are also discussed. Chapter 4, by Vestroni, deals with the analysis of structural identification techniques based on parametric models. A numerical code, that implements a variational procedure for the identification of linear finite element models based on modal quantities, is presented and applied for modal updating and damage detection purposes. Pseudo-experimental and experimental cases are solved. Ill-conditioning and other peculiarities of the method are also investigated. Chapter 5, by Vestroni, deals with damage detection in beam structures via natural frequency measurements. Cases of single, multiple and interacting cracks are considered in detail. Attention is particularly focussed on the consequences that certain peculiarities, such as the limited number of unknowns (e.g., locations and stiffness reduction of damaged sections), have on the inverse problem solution. The analysis of damage identification in vibrating beams is continued in Chapter 6 by Morassi. Damage analysis *is formulated as a reconstruction problem and it is shown that frequency shifts caused by damage contain information on certain Fourier coefficients of the unknown stiffness variation. The rest of the chapter is devoted to the identification of localized damage in beams from a minimal set of natural frequency measurements. Closed form solutions for certain crack identification problems in vibrating rods and beams are presented. Applications based on changes in the nodes of the mode shapes and on antiresonant data are also discussed. Chapter 7, by Testa, is on the localization of concentrated damage in beam structures based on frequency changes caused by the damage. A second application deals with a crack closure that may develop in fatigue and the potential impact on damage detection. Chapter 8 proposes a paper by Cawley on the use of guided waves for long-range inspection and the integrity assessment of pipes. The aim is to determine the reflection coefficients from cracks and notches of varying depth, circumferential and axial extent when the fundamental torsional mode is travelling in the pipe. Chapter 9, by Vestroni and Vidoli, discusses a technique to enhance sensitivity of the dynamic response to local variations of the mechanical characteristics of a vibrating system based on coupling with an auxiliary system. An application to a beam-like structure coupled to a network of piezoelectric patches is discussed in detail to illustrate the approach.*

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CONTENTS

Elements of Experimental Modal Analysis

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Abstract Fundamental concepts for the characterization of the dynamical response of SDOF and NDOF systems are provided. A description is given of the main techniques to represent the response in the frequency domain and its experimental characterization. Two classical procedures of modal parameter identification are outlined and selected numerical and experimental examples are reported.

1 Dynamic Characterization of a SDOF

The experimental study of a structure provides an insight into the real behavior of the system. In particular, the study of its dynamic response, exploiting vibration phenomena, aims to determine the dynamic properties closely connected to the geometrical and mechanical characteristics of the system. Hence, some concepts of structural dynamics will be briefly summarized. It is assumed that the reader has had some exposure to the matter (Craig, 1981; Meirovitch, 1997; Ewins, 2000; Braun et al., 2001).

The classical model of a single degree-of-freedom (SDOF) system is the spring-mass-dashpot model of Figure 1, where the equation of motion and the steady-state solution is reported. Assuming a harmonic excitation, the frequency response function (FRF) $H(\omega)$ can be defined as the ratio between the amplitude of the steady-state response and the load intensity. The FRF shows that in a small range of the ratio ω/ω_0, when the frequency of the excitation approaches the natural frequency of the system, the response amplitude is much larger than the static response. This is called resonance. Furthermore, the amplitude of the steady-state response is linearly dependent on both p_0 and $H(\omega)$. By knowing $H(\omega)$ the response of a SDOF system to a harmonic excitation can be estimated.

In the real world, forces are not simply harmonic, being frequently periodic or approximated closely by periodic forces. A periodic function $p(t)$ having period T_1 can be represented as a series of harmonic components by means of its Fourier series expansion. As an example, in Figure 2, the Fourier series expansion is applied to a square wave. The Fourier series is convergent, i.e. the more terms used, the better the approximation obtained.

Since the response of a SDOF system to a harmonic force is known and a periodic forcing function $p(t)$ can be represented as a sum of harmonic forces, the response of the system $u(t)$ to a periodic excitation can be obtained by exploiting the principle of effect superposition:

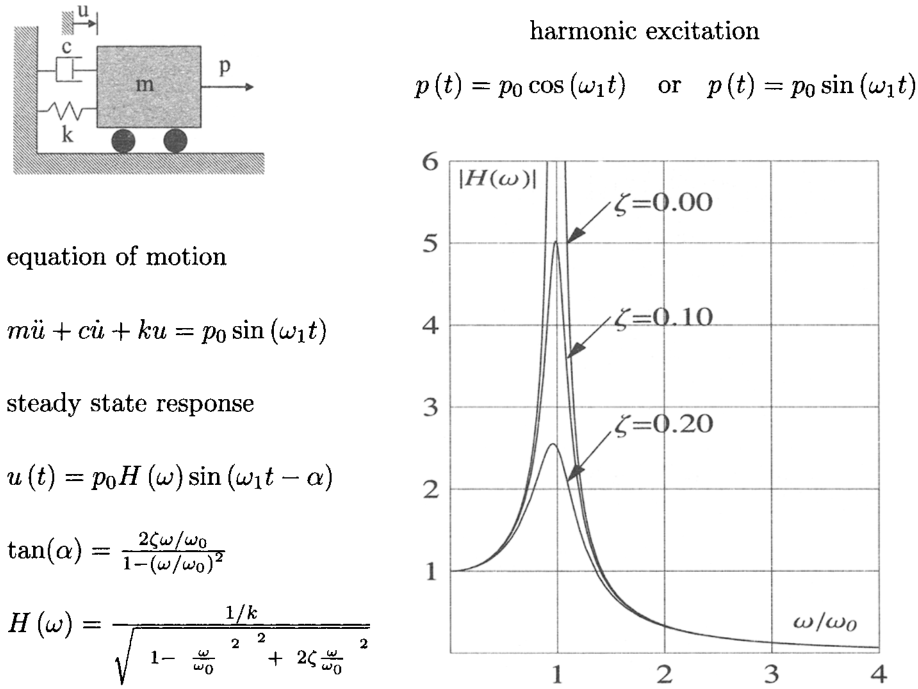


Figure 1. Response of a SDOF system to a harmonic excitation.

$$p(t) = \sum_{n=1}^{\infty} p_n \cos(\omega_n t + \varphi_n), \quad \omega_n = n\omega_1 \tag{1.1}$$

$$u(t) = \sum_{n=1}^{\infty} U_n \cos(\omega_n t + \varphi_n - \alpha_n) \tag{1.2}$$

$$U_n = \frac{p_n/k}{\sqrt{(1-r_n^2)^2 + (2\zeta r_n)^2}} = p_n H(\omega_n, \zeta), \quad tg\alpha_n = \frac{2\zeta r_n}{1-r_n^2}, \quad r_n = \frac{\omega_n}{\omega_0}. \tag{1.3}$$

In this case, too, a knowledge of $H(\omega)$ is sufficient to predict the response of the system.

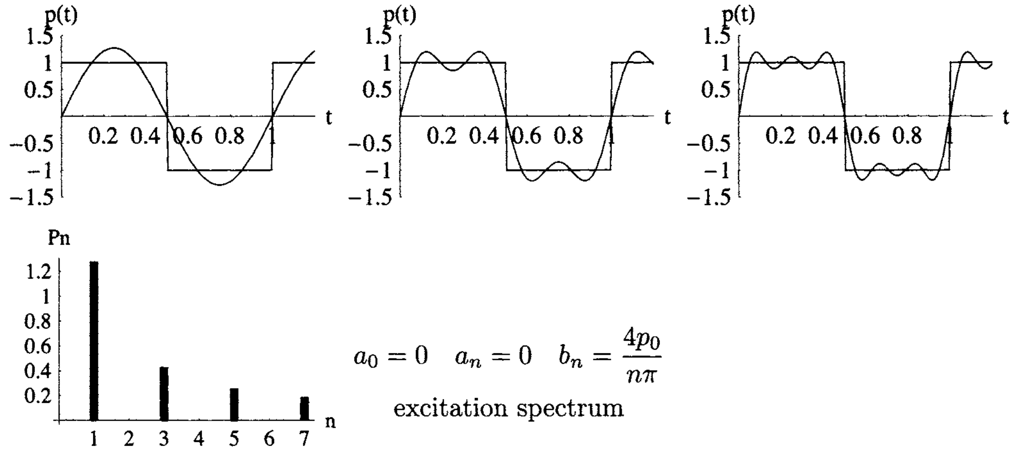
The steady-state response of a SDOF system to a harmonic force can also be written in complex form, where the bar denotes complex quantities:

$$\overline{u}(t) = \overline{U}(\omega) e^{i\omega t} = \overline{H}(\omega) p_0 e^{i\omega t} \quad \text{and} \quad \overline{H}(\omega) = \frac{1/k}{\left[1-(r)^2\right] + i(2\zeta r)}. \tag{1.4}$$

It is clear that the amplitude and phase of the steady-state response are determined from the amplitude and phase of the complex FRF.

periodic excitation $T_1\,,\omega_1 \quad p\,(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos\left(n\omega_1 t\right) + \sum_{n=1}^{\infty} b_n \sin\left(n\omega_1 t\right)$

example: square wave $T_1 = 1 \quad p\,(t) = \sum_{n=1,3}^{\infty} b_n \sin\left(n\omega_1 t\right)$ odd function



$$a_0 = 0 \quad a_n = 0 \quad b_n = \frac{4p_0}{n\pi}$$

excitation spectrum

Figure 2. Fourier series expansion of a square wave.

If the forcing function is periodic, the response can be written as:

$$\overline{u}\,(t) = \sum_{n=-\infty}^{\infty} \overline{U}_n\, e^{in\omega t} = \sum_{n=-\infty}^{\infty} \overline{H}\,(\omega_n)\,\bar{p}_n\, e^{in\omega t}. \tag{1.5}$$

When the excitation is non periodic, it can be represented by a Fourier integral, which is obtained from the Fourier series by letting the period T_1 approach infinity. Let us define:

$$T_1 = \frac{2\pi}{\omega_1}\,, \quad \omega_1 = \Delta\omega\,, \quad n\omega_1 = \omega_n. \tag{1.6}$$

In the Fourier series

$$p\,(t) = \sum_{n=-\infty}^{\infty} \overline{p}_n\,(\omega_n)\, e^{i\omega_n t}\,, \quad \overline{p}_n\,(\omega_n) = \frac{1}{T_1}\int_{T_1} p\,(t)\, e^{-i\omega_n t} dt \tag{1.7}$$

since T_1 tends to infinity, p_n is newly defined as:

$$\overline{p}_n\,(\omega_n) = T_1 \overline{p}_n\,(\omega_n) \quad \text{and} \quad p\,(t) = \sum_{n=-\infty}^{\infty} \frac{\Delta\omega}{2\pi}\overline{p}_n\,(\omega_n)\, e^{in\Delta\omega t}. \tag{1.8}$$

When $T_1 \longrightarrow \infty$, $n\Delta\omega = \omega_n = \omega$ becomes a continuous variable and $\Delta\omega$ becomes the differential $d\omega$, then a Fourier transform pair is obtained:

$$\bar{P}(\omega) = \int_{-\infty}^{\infty} p(t)e^{-i\omega t}dt \quad \text{direct Fourier transform} \tag{1.9}$$

$$p(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi}\overline{P}(\omega)e^{i\omega t}d\omega \quad \text{inverse Fourier transform.} \tag{1.10}$$

When the forcing function is non periodic, a relationship in the frequency domain between the response and the force can be obtained by applying the Fourier transform to each term of the motion equation:

$$\left(-m\omega^2 + ic\omega + k\right)\bar{U}(\omega) = \bar{P}(\omega) \tag{1.11}$$

where use is made of the following properties:

$$\dot{\bar{U}}(\omega) = \frac{i\omega}{2\pi}\int_{-\infty}^{\infty} u(t)\,e^{-i2\pi ft}dt = \frac{i\omega}{2\pi}\bar{U}(\omega) \tag{1.12}$$

$$\ddot{\bar{U}}(\omega) = \frac{-\omega^2}{2\pi}\int_{-\infty}^{\infty} u(t)\,e^{-i2\pi ft}dt = \frac{-\omega^2}{2\pi}\bar{U}(\omega)\,. \tag{1.13}$$

The Fourier transform of the response is obtained as the product of the complex FRF and the Fourier transform of the excitation

$$\bar{U}(\omega) = \bar{H}(\omega)\,\bar{P}(\omega)\,. \tag{1.14}$$

Once $U(\omega)$ is known, the response in the time domain is given by the inverse Fourier transform:

$$\overline{u}(t) = \int_{-\infty}^{\infty} \frac{1}{2\pi}\overline{U}(\omega)\,e^{i2\pi ft}df. \tag{1.15}$$

In this case also, by knowing $H(\omega)$, the response to a generic excitation can be estimated.

A significant relationship exists between the $H(\omega)$ and the unit impulse response $h(t)$. The latter defines the SDOF response in the time domain to a forcing function equal to a Dirac delta. Since the Fourier transform of an impulse $p(t)=\delta(0)$ is

$$p(\omega) = \frac{1}{2\pi}, \tag{1.16}$$

the impulse response can be written as

$$u(t) = h(t) = \int_{-\infty}^{\infty} \overline{H}(\omega)\,\overline{p}(\omega)e^{i\omega t}d\omega = \frac{1}{2\pi}\int_{-\infty}^{\infty} \overline{H}(\omega)e^{i\omega t}d\omega. \tag{1.17}$$

In other words, the time domain response to a Dirac delta is the inverse Fourier transform of the FRF. The FRF and the impulse response function form a couple of Fourier transforms. It is possible to refer both to $H(\omega)$ or to $h(t)$ to characterize the system and to provide a predictive model.

2 Display of a FRF

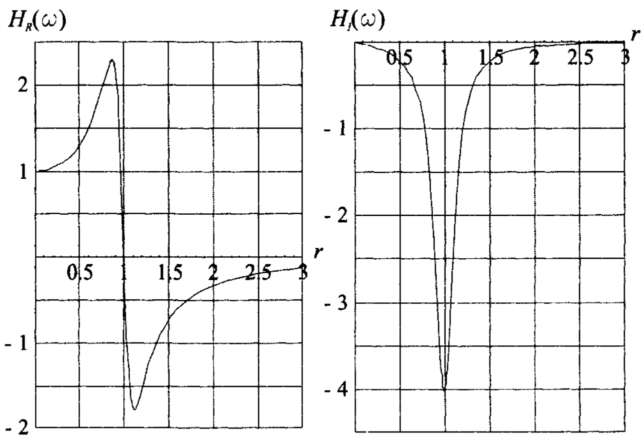
As a complex quantity, the FRF contains information regarding both the amplitude and the phase of the oscillation.

The real and imaginary components of $H(\omega)$ are:

$$H_R(\omega) = \frac{\left(1 - r^2\right)/k}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2} \quad H_I(\omega) = \frac{-2\zeta r/k}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}. \tag{2.1}$$

The three most common forms of representation of $H(\omega)$ are reported in Figure 3-4 for a SDOF with $\zeta = 0.125$.

(1) Real and Imaginary parts of $H(\omega)$ vs r (Figure 3). The real part of $H(\omega)$ crosses the frequency axis at resonance, while, at the same frequency, the imaginary part reaches a minimum.



$$H_R(\omega) = \frac{1 - r^2/k}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}$$

$$H_I(\omega) = \frac{-2\zeta r/k}{\left(1 - r^2\right)^2 + \left(2\zeta r\right)^2}$$

Figure 3. Real and Imaginary part of the FRF vs r.

(2) Modulus of FRF and phase vs r (Figure 4). Resonance is pointed out by a maximum in the modulus of the FRF and by a phase change from 0 to $-\pi$.

(3) Real part vs Imaginary part in the Argand plane (Figure 4). This is a circular loop that contains all the information and enhances the region close to resonance, which is practically coincident with the intersection of the circle with the y axis.

The dynamic properties of a system can be expressed in terms of any convenient response characteristics: FRF can be presented in terms of displacement (receptance), velocity (mobility) or acceleration (intertance). Mobility and inertance are obtained from receptance by multiplying by $i\omega$ and $(i\omega)^2$, respectively.

From the analytical relationships, previously reported, it is clear that the FRF can be experimentally evaluated mainly by two different methods (Ewins, 2000; Maia and Silva, 1997). The former involves the steady-state response to a harmonic force P at different assigned frequencies, which implies the use of an exciter connected to the structure that