Fields Institute Communications 67

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Arithmetic and Geometry of K3 Surfaces and Calabi-Yau Threefolds

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Preface

The workshop on *Arithmetic and geometry of K3 surfaces and Calabi–Yau threefolds* was held at the Fields Institute and University of Toronto from August 16 to 25, 2011. The workshop was organized by Charles F. Doran (Alberta), Shigeyuki Kondō (Nagoya), Radu Laza (Stony Brook), James D. Lewis (Alberta), Matthias Schütt (Hannover), and Noriko Yui (Kingston/Fields).

This proceedings volume for the 2011 Calabi–Yau workshop is edited by Radu Laza, Matthias Schütt, and Noriko Yui. The editors wish to express their appreciation to all the contributors for preparing their manuscripts for the Fields Communications Series, which required extra effort in presenting not only current developments but also some background material on the discussed topics. All papers in this volume were peer-reviewed. We are deeply grateful to all the referees for their efforts in evaluating the articles, in particular, in the limited time frame.

The workshop was financially supported by various organizations. In addition to the Fields Institute, the workshop was supported by NSF (grant no. 1100007), JSPS (Grant-in-Aid (S), No. 22224001), and DFG (GRK 1463 "Analysis, Geometry and String Theory"). Additionally, several participants have used their individual grants (e.g., NSF or NSERC) to cover their travel expenses. We are thankful to all these sponsor organizations: their support made possible the participation of a large number of junior participants and of a significant number of researchers from outside North America. This in turn led to a very dynamic and active workshop.

Some of the articles were copy-edited by Arthur Greenspoon of Mathematical Reviews. The editors are grateful for his help towards improving both the stylistic and mathematical presentations.

Last but not least, we thank Debbie Iscoe of the Fields Institute for her help in reformatting articles in the Springer style and assembling this volume for publication.

Stony Brook, NY — Radu Laza
Hannover, Germany — Matthias Schütt
Kingston, ON — Noriko Yui

Introduction

In recent years, research in K3 surfaces and Calabi–Yau varieties has seen spectacular progress from both arithmetic and geometric points of view, which in turn continues to have a huge influence and impact on theoretical physics, in particular, on string theory. The workshop was designed to bring together experts and junior researchers who are aspiring to become experts for 10 days at the Fields Institute in August 2011 to review recent developments, inspire graduate and postdoctoral fellows and young researchers, and also explore future directions of the subjects. With 114 (officially registered) participants, there was a wide geographical representation, with a very significant presence of European and Japanese participants (in addition to US and Canadian participants).

The workshop started with a 3-day introductory session aimed at graduate students and postdoctoral fellows, followed by a 1 week research conference with Sunday off. The introductory lectures were intended to give some background and a brief overview of the vast topic of Calabi–Yau varieties and K3 surfaces. At the subsequent research conference, there were in total 35 research talks presented on wide ranges of topics around K3, Enriques and other surfaces, and Calabi–Yau threefolds and higher-dimensional varieties and manifolds, some of which can be found in this volume.

As a consequence of the significant interest in the subject, we are organizing a follow-up extended concentration period on Calabi–Yau varieties, in the form of a semester long thematic program *Calabi–Yau varieties: arithmetic, geometry and physics* at the Fields Institute (July to December 2013). This thematic program is devoted to the arithmetic and geometry of Calabi–Yau varieties and the connections to physics, especially string theory.

Scientific Focus of the Workshop

The workshop concentrated on topics (on the geometry and arithmetic of Calabi–Yau varieties) that have either seen great progress recently or shown a high potential for future inventions. Specifically, the major topics covered included:

1. Families and degenerations of Calabi–Yau varieties—moduli theoretic and arithmetic viewpoints.
2. Modularity: Galois representations of Calabi–Yau varieties and their connections to automorphic forms, in particular to classical, Hilbert, and Siegel modular forms.
3. Calabi–Yau varieties of CM type and with special automorphisms, especially K3 surfaces with symplectic and non-symplectic automorphisms.
4. Algebraic cycles and motives: divisors, CM cycles, and motives arising from K3 surfaces and Calabi–Yau threefolds.
5. Variations of mixed Hodge structures, periods, and Picard–Fuchs differential equations.

Overview of This Volume

In the following paragraphs we give a brief overview of the volume. There are in total 24 articles. Some of the articles are written-up versions of the talks presented at the workshop, while others report on subsequent developments on the subject matter of the workshop. Roughly the articles are divided into three categories, namely:

- *Introductory lectures*
- *Arithmetic and geometry of K3, Enriques and other surfaces*
- *Arithmetic and geometry of Calabi–Yau threefolds and higher dimensional varieties*

The workshop's program contained several other talks on related topics. Additional documentation from the workshop is available on the homepage maintained by the Fields Institute.[1]

Introductory Lectures

There are four survey papers by Kondō, Lewis, Schütt, and Yui which comprise a selection of the lectures given by the organizers during the 3-day introductory period of the workshop. These lectures were mostly aimed at junior participants of the workshop and often geared specifically towards some of the talks to be given

[1] http://www.fields.utoronto.ca/programs/scientific/11-12/CalabiYau/index.html.

during the research conference. Similarly for this volume, the survey papers can be used as a starting point or as a guide to the subject.

The surveys by Kondō and Schütt review the geometry and arithmetic of K3 surfaces, including basics such as lattice theory. Yui's paper presents the current status on modularity of Calabi–Yau varieties in its different incarnations. The focus lies on Calabi–Yau varieties of dimension at most three. Lewis reviews transcendental aspects of algebraic cycles and, specializing to the Calabi–Yau situation, explains some recent developments in the field.

Arithmetic and Geometry of K3, Enriques, and Other Surfaces

A common theme of many papers in this section are elliptic fibrations. Most notably, Bertin and Lecacheux classify all elliptic fibrations on a specific K3 surface. From a similar elliptic modular surface, Anema and Top derive explicit algebraic coverings of a pointed torus. On the moduli side, Besser and Livné relate specific elliptic K3 surfaces to abelian surfaces with quaternionic multiplication; this produces explicit Shimura curves. Special cycles in moduli spaces of lattice polarized K3 surfaces are treated by Kudla. These higher Noether–Lefschetz loci are the input of certain generating series whose modularity is known.

Elliptic fibrations form also the key ingredient for Kerr's approaches to the computation of transcendental invariants of indecomposable algebraic K_1 classes. Moreover Kerr's work builds on toric geometry which features prominently in the paper of Whitcher et al. as well. Here three-dimensional reflexive polytopes with S_4 symmetry are related to natural one-parameter family of K3 surfaces with symplectic S_4 action. Picard–Fuchs equations are studied not only in this paper but also by Gährs for certain one-parameter families associated with invertible polynomials, using the GKZ system.

Oguiso's paper is concerned with a classical problem: it proves that there is a smooth quartic K3 surface automorphism that is not derived from a Cremona transformation. Almost as classical a problem for Enriques surfaces, Dolgachev extends results for cohomologically or numerically trivial automorphisms to arbitrary characteristics. Contrary to previous approaches, the key tool is Lefschetz' fixed point formula. Enriques surfaces of Hutchinson–Göpel type are investigated by Mukai and Ohashi. Starting from the projective geometry of Jacobian Kummer surfaces, they give a sextic presentation for these Enriques surfaces and then describe their intrinsic symmetries.

On the less classical side, Hulek and Ploog extend the theory of Fourier–Mukai partners to include the presence of polarizations, producing a counting formula for the number of partners. In a different direction, Schoen's paper computes invariants of the degeneration of a product of elliptic curves upon split-muliplicative reduction. He expatiates on divisor class group, (co)homology, and Picard group of the closed fibers. To close the circle on fibrations, Heijne and Kloosterman study a special class of surfaces, the so-called Delsarte surfaces. Singled out by their accessibility to explicit computations, a classification of some specific fibrations on these surfaces is given.

Arithmetic and Geometry of Calabi–Yau Threefolds and Higher Dimensional Varieties

For one-dimensional families of Calabi–Yau manifolds, Cynk and van Straten compute Picard–Fuchs operators based on the expansion of a period near a conifold point. The algorithm is explained in detail and illustrated by some concrete examples consisting in double octics. In contrast, the paper by Gouvêa, Kiming, and Yui considers rigid Calabi–Yau threefolds defined over $\mathbf{Q}$. Motivated by the geometric realization problem they pose the question of whether the Calabi–Yau threefolds admit quadratic twists, giving answers for a number of examples. Automorphic forms are also the main players in the papers by Kondō and Movasati. Kondō uses Borcherds theory of automorphic forms on orthogonal groups to construct a rational map from the Segre cubic threefold to its dual, the Igusa quartic threefold. The major novelty of Movasati's paper consists in a modification of the interplay between moduli of polarized Hodge structures of a fixed type and Griffiths period domains. While in the classical case (for Hermitian symmetric domains) one obtains automorphic forms and algebraic structures on the mentioned moduli spaces, Movasati's formulation leads to a notion of quasi-automorphic forms.

The foundations for Donaldson–Thomas invariants for stable sheaves on algebraic threefolds with trivial canonical bundle are reviewed in Gulbrandsen's contribution. Special emphasis lies on abelian threefolds. Chen and Lewis prove density statements about the subgroup of invertible points on intermediate Jacobians. They focus on the points in the Abel–Jacobi image of nullhomologous algebraic cycles on projective algebraic manifolds. Last but not least, the paper by Pearlstein and Schnell concerns the infinitesimal invariant of a normal function on a complex manifold. When the manifold is quasi-projective and the function is admissible, they show that this zero locus is constructible in the Zariski topology.

Acknowledgment

Let us conclude by expressing our sincere thanks to all the contributors to this volume, the referees, the sponsoring institutions, and, most of all, all participants of the workshop for creating the stimulating atmosphere from which this volume arose.

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