CHAPTER 1

MATHEMATICS AND CULTURE

The idea that mathematical objects are in some sense eternal and independent of the flux of history and culture has its roots in the ancient worlds of Pythagoras and Plato. It has survived into modern times, and in one form or another has played a role in the thinking of most students of knowledge and science. The resistance to a sociology of mathematics has not, however, always rested on the naive notion of a "real" Platonic realm of Ideals. It has often stemmed from a fear of or resistance to the relativistic implications of any sociology of knowledge. But the conception of mathematics as a social fact does not entail relativism. Resolving the apparent contradiction between the fact of a recalcitrant reality and the idea that reality is socially constructed requires seeing such notions as mind, consciousness, knowledge, and nature in a new way. One of my objectives is to point in the direction of just such a new perspective. But I will be cautious about claiming that this still embryonic perspective is transparent.

Neither relativists nor realists will find support for their viewpoints in my explorations. The realists may be ready to claim me as a comrade when I announce that I am no enemy of the real world, truth, and objectivity. But the relativists are just as likely to embrace me when I argue that truth, and objective statements are shaped by human hands and brains in arenas of social production. I expect that in the course of reading this book readers will develop an appreciation for, if not a transparent understanding of, the idea that all of our thoughts and actions are social constructs. In any case, however, I ask readers to keep in mind that when I argue that mathematics is social through and through I do not mean that it is somehow "arbitrary" or "random".

A natural starting point for any sociology of mathematics is Oswald Spengler's thesis that each "Culture" has its own conception of number. This is the most dramatic expression of an idea adumbrated in Emile Durkheim's reflections on logical concepts as collective representations, and mirrored in various forms in the ruminations on the "anthropology" of mathematics by Ludwig Wittgenstein and others. It is not necessary to endorse Spengler's concept of the "soul" of a civilization and associated metaphysical postures (often misunderstood, in any case), nor his brand of nationalism (often incorrectly interpreted as "Hitlerian") in order to appreciate his uncompromising explanatory and materialist approach to mathematics.

Spengler's discussion of numbers occurs prominently in Chapter Two of the first volume of *The Decline of the West.* He chooses *number* "to exemplify the way in which a Soul seeks to actualize itself in the picture of its outer world – to show, that is, in how far culture in the 'become' state can express or portray an idea of human existence" (Spengler, 1926: 56). *Number*, "the primary element on which all mathematics rests", is *specifically* chosen "because mathematics, accessible in its full depth only to the very few, holds a quite peculiar position amongst the creations of the mind". Mathematics is "peculiar" because it is simultaneously a "science" ("fuller" and "more comprehensive" than logic), a "true art", and a "metaphysic". There is no a priori reason to agree with Spengler that mathematics is unique in this three-fold way; most if not all students of the sociology or natural history of mathematics assume mathematics is in some way unique among modes of knowing. What is significant is that Spengler's analysis is a formidable attack on the privileged status of mathematics as an intellectual or scholarly discipline. Before proceeding further, it will prove useful to briefly review some of the basic terms Spengler uses in his analysis of history.

Spengler makes several axiomatic distinctions: (1) "becoming" and "the become" (roughly, "process" and "result", or "experience as lived" and "experience as learned"); (2) "alien" (the outer world of sensation) and "proper" (the inner life of feeling); (3) "soul" (the possible, the future, "the still to be accomplished") and "world" (the actual, the past, "the accomplished"); and (4) "Nature" ("the numerable") and "History" ("everything unrelated to mathematics"). Spengler (1926: 59) writes:

> An actuality is Nature in so far as it assigns things-becoming their place as things-become, and History in so far as it orders things-become with reference to their becoming.

"Life" is "the form in which the actualizing of the possible is accomplished" (Spengler, 1926: 59). These ideas are introduced as part of "an immediate inward certainty", that is, basic or elemental facts of consciousness. Waking-consciousness is conceived "structurally" as a "tension of contraries" (Spengler, 1926: 54). These contraries share two important features: (1) they are each units or totalities (and together they form a totality), and (2) they are polarities which by virtue of being extremes establish that there is a potential for many types of "realities" (Spengler, 1926: 55):

> The possibilities that we have of possessing an "outer world" that reflects and attests our proper existence are infinitely numerous and exceedingly heterogeneous, and the purely organic and the purely mechanical worldview ... are only the extreme members of the series.

Finally, it is important to understand that Spengler uses Culture in a specific sense, a sense different from that associated with the anthropological concept of culture. When Spengler claims that "primitive man" has no Culture, he means that a "real knowledge of history and nature" is lacking. Only when the ensemble of self, history, and nature becomes separated for the waking-consciousness can we speak of Culture (Spengler, 1926: 55). In much the same way, Marx distinguished all human activity up to the threshold of communism as "prehistory", and communism as the beginning of "human history". This does not mean that there has not been any "history" in the conventional sense; nor does Spengler mean that there are human societies without "culture".

In order to appreciate Spengler's notion of number, it is important to understand that he conceives of a "fundamental connexion between *the become* (*the hard set*) and *Death*" (Spengler, 1926: 54). He then argues (Spengler, 1926: 56–57):

> The real secret of all things-become, which are *ipso facto* things extended (spatially and materially), is embodied in mathematical number as contrasted with chronological number. Mathematical number contains in its very essence the notion of a *mechanical demarcation*, number being in that respect akin to *word*, which, in the very fact of its comprising and denoting, fences off world-impressions.
> In number, then, as the *sign of completed demarcation*, lies the *essence* of everything actual, which is cognized, is delimited, and has become all at once – as Pythagoras and certain others have been able to see with complete inward certitude by a mighty and truly religious intuition.

The connection between mathematics and religion suggested in Spengler's reference to the religious intuition of Pythagoras is implied in the conception of mathematics as a world view.

Number, according to Spengler, is "the symbol of causal necessity". Number and the conception of God both contain "the ultimate meaning of the world-as-nature". The deep affinity between religion and mathematics is clearly evident in the Pythagoreans and the Platonists. But it is also present in Descartes, Pascal, and Leibniz (Spengler, 1926: 66). Religious intuition, Spengler argues, is behind the great mathematical discoveries of the greatest mathematical thinkers – "the creative artists of the realm of numbers" – in all Cultures. These people who experience the spirit of number living within themselves realize that they "know God"; Number is akin to God, and it is related to myth insofar as it originated in the "naming process" associated with the will to "power over the world" (Spengler, 1926: 56–57). There is a clear rationale for Spengler's conjecture on the relationship between mathematics and religion in the cases he cites as well as in such cases as the relationship between the medieval discourses on infinity motivated by theological questions and the development of the calculus, the differences between early British and modern algebra, and the relationship between theology and mathematics in the works of Boole, Cantor, and others. These cases will be discussed later in this book.

Cultures, according to Spengler, are incommensurable. Our present minds, he argues, are "differently constituted" than minds in earlier Cultures. Therefore, earlier mathematical events should not be viewed as stages in the development of "Mathematics" (Spengler, 1926: 57). The two major Cultures Spengler identifies are Classical and Western. He also identifies two minor Cultures: Babylonian-Egyptian and Arabian-Islamic (Indian and Chinese Cultures are also recognized in his schema). Each major Culture experiences the same birth-death sequence in its number-world: (1) *conception* of a new number form, (2) *zenith* of systematic development, and (3) *inward completion and conclusion* of a figure-world. In Classical Culture, the sequence is: (1) the Pythagorean conception of number as "magnitude"; (2) the achievement of the zenith between 450BCE and 350BCE in the works of Plato, Archytas, and Eudoxus; and (3) the inward completion in the works of Euclid, Apollonius, and Archimedes between 300BCE and 250BCE. In Western Culture, the sequence is: (1) the conception of number as "relation" (Descartes, Pascal, Fermat, Newton, Leibniz) in the seventeenth century; (2) the zenith achieved by Euler, Lagrange, and Laplace (1750–1800); and (3) the inward completion achieved from 1800 onwards by Gauss, Cauchy, and Riemann. Let us examine these differences in more detail.

Classical mathematics deals with number as magnitude, as the essence of what can be perceived through the senses, that is, viable, tangible units.

It is confined to facts in the present that are near, and small, with a focus on the properties of individual entities and their boundary surfaces (stereometry, or solid geometry). In general, it is confined to positive and whole numbers, and proportion as the nexus of magnitude. Western mathematics liberates geometry from the visual and algebra from magnitude. Numbers are images of "pure thought" (or "desensualized understanding"), and their abstract validity is self-contained. The focus is on whole classes of formal possibilities, groups of functions, and other *relations*; function is the nexus of relations. Whereas Classical mathematics affirms appearances, Western mathematics denies them; thus the opposition between fear of the irrational in Classical mathematics and the central role of the analysis of the infinite in Western mathematics. In Classical mathematics, the straight line is a measurable edge; in Western mathematics it is an infinite continuum of points – indeed, the core unit of Western mathematics is, Spengler argues, the "abstract space-element of the point", and the main theoretical objective is the interpretation of *space* (a "great and wholly religious symbol", in Spengler's view). Whereas enlargements and reductions of scale and the constancy of constituents are characteristic of Classical mathematics, Western mathematics is based on group transformations and the variability of constituents. In Classical mathematics, the equality sign in

$$3^2 + 4^2 = 5^2$$

establishes a rigid relationship between specific amplitudes, and signals that a problem is being worked out to a result. In Western mathematics

$$X^n + Y^n = Z^n$$

is Classical in appearance but is in reality a new kind of number. It is a picture and sign of a relation – the equality sign does not point to a result in the Classical sense (and because of this, Spengler argues, a new symbolism is needed in order to eliminate the vestigial and confusing parallels with Classical mathematics).

Spengler's characterization of Classical and Western polarizes differences and so underscores his view of the incommensurability between the Greek concept of number and the concept(s) of number fashionable among professional mathematicians from the late nineteenth century onward. But when Spengler contrasts Classical and Western, the latter label refers already to Descartes, Pascal, Fermat, and Leibniz. Earlier, the linkage between Classical and Western was much stronger.